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Technical Note

Effect of viscous dissipation on forced-convection heat transfer in cylindrical packed-beds

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1. Introduction

Relating to thermal design of regenerative heat exchangers, catalytic converters, and adsorption beds, modeling of forced-convection heat transfer in porous media is an important research subject in thermal engineering. Numerous experimental researches, theoretical analyses and numerical simulations have been made during past few decades. At the present time, Darcy–Brinkman–Forchheimer model simultaneously considering the inertia and boundary effects [1] with the radial porosity variations [2,3] is adopted as the momentum equation. Furthermore, in the energy equation, the effects of porosity-dependent radial thermal dispersion, variable effective thermal conductivity and thermal radiation [4–7] are taken into account. Inclusion of these effects in the governing equations has greatly improved the porous media models and these effects in porous media were verified to a certain extent by various experimental data. However, these models still do not account for all of the physical phenomena that may exist in porous media.

Almost all previous theoretical studies are based on the hypothesis that the effect of heat generation due to viscous dissipation in porous media on heat transfer characteristics is negligible. The viscous dissipation term in an empty tube can be confidently neglected for most cases. In porous media, however, a solid–fluid contact area is many times greater than the duct surface area. Then, is it a correct assumption for all flow conditions? The purpose of the present note is to examine the validity of the premise that the effect of viscous dissipation on forced-convection heat transfer in packed beds can be neglected. To this end, first, we extend the previous heat transfer model [6] by taking into account the effect of viscous dissipation in the energy equation. And then we solve the governing equations numerically under adiabatic and isothermal boundary conditions. The results are compared with available experimental data.

2. Analysis

The following assumptions are introduced for the analyses:

1. the fluid and solid phases are in local thermal equilibrium;
2. the flow field is hydrodynamically fully developed at the inlet of the packed bed;
3. the fluid enters the bed with a uniform inlet temperature T_0;
4. the wall of the bed is adiabatic or is kept at constant temperature;
5. the fluid is liquid such that the relevant physical properties except for the volumetric expansion coefficient do not depend on temperature;
6. local porosity within the packed bed varies with distance from the boundaries alone;
7. the boundary-layer approximation is valid.

Under these assumptions, the steady-state continuity and momentum equations for forced-convection in variable porosity media are

$$\frac{\partial u}{\partial x} = 0, \tag{1}$$

$$-\frac{\mathrm{d}p}{\mathrm{d}x} = \frac{\mu}{K}u + \rho C u^2 - \frac{\mu}{\phi}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right). \tag{2}$$

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Nomenclature

B	βT_0 for the adiabatic wall and βT_w for the isothermal wall
C	inertial coefficient $(=1.75(1-\phi)/d_p\phi^3)$
C^*	C/C_∞
c_p	specific heat (J/kg K)
Da	Darcy number $(=K_\infty/r_0^2)$
d_p	particle diameter (m)
E_u^*	dimensionless pressure gradient defined by $-(\mathrm{d}p/\mathrm{d}x)/(\rho u_m^2/2r_0/RePr)$
E_k^*	modified Eckert number defined by $(v_f/2r_0)^2/c_pT_0$ for the adiabatic wall and by $(v_f/2r_0)^2/c_pT_w$ for the isothermal wall
F_h	Forchheimer number $(=C_\infty r_0)$
K	permeability $(=d_p^2\phi^3/150(1-\phi)^2)$
K^*	K_∞/K
k_d	thermal dispersion conductivity of a packed bed (W/m K)
k	thermal conductivity (W/m K)
Nu_m	mean Nusselt number defined in Eq. (12)
Pr	Prandtl number of fluid $(=\mu c_p/k_f)$
Pec	Peclet number $(=RePr)$
Re	Reynolds number based on the diameter of a packed column $(=2u_m r_0/v)$
r	radial coordinate
r_0	radius of a cylindrical packed-column (m)
T	temperature (K)
u	axial velocity (m/s)
U	dimensionless axial velocity $(=u/u_m)$
x	axial coordinate

Greek symbols

Γ	ratio of the radius of a packed column to the particle diameter $(=r_0/d_p)$
η	dimensionless radial coordinate $(=r/r_0)$
θ	dimensionless temperature defined by T/T_0 for the adiabatic wall and by T/T_w for the isothermal wall
κ	ratio of thermal conductivity of the solid to that of the fluid $(=k_s/k_f)$
ξ	dimensionless axial coordinate $(=x/2r_0/(RePr))$
v	kinematic viscosity of fluid $(\mathrm{m}^2/\mathrm{s})$
μ	viscosity of fluid (Pa s)
ρ	density of fluid $(\mathrm{kg/m}^3)$
ϕ	porosity

Subscripts

e	effective
f	fluid
s	solid
m	mean
w	wall
0	inlet
∞	quantity at $\phi=0.39$

The energy equation may be written as

$$\rho c_p u\frac{\partial T}{\partial x}=\frac{1}{r}\frac{\partial}{\partial r}\left[r(k_e+k_d)\frac{\partial T}{\partial r}\right]+\Phi+\beta Tu\left(\frac{\mathrm{d}p}{\mathrm{d}x}\right). \qquad (3)$$

The dissipation term Φ [8] represents the mechanical power needed to extrude the viscous fluid through the pore and is equal to the flow rate times the externally maintained pressure drop $[=u(-\mathrm{d}p/\mathrm{d}x)]$, provided that the kinetic energy remains constant along the flow direction. The third term of the right-hand side denotes the amount of heat absorbed by a volume element due to thermal expansion. Although this term is small enough compared with Φ, it cannot be neglected in considering the effect of viscous dissipation in a porous medium. Because, in the case of ideal gases, β becomes $1/T$ and the second and third terms of the right-hand side are cancelled each other: this is fully consistent with the fact that the Joule–Thomson expansion of ideal gases in a constant enthalpy process does not yield any temperature change.

Moreover, it is of interest to note that, in Eq. (3), the effect of turbulence in a packed bed is empirically taken into account in form of the thermal dispersion conductivity.

The corresponding boundary conditions are

$$r=0:\ \partial u/\partial r=\partial T/\partial r=0,\ r=r_0:\ u=0,\ \partial T/\partial r=0 \quad \text{for the adiabatic wall or} \quad T=T_w \quad \text{for the isothermal wall.} \qquad (4)$$

Introducing the dimensionless quantities defined in the nomenclature yields the governing equation of the form

$$\frac{E_u^*}{4Pr}=\frac{K^*U}{Da}+\frac{F_hRe}{2}C^*U^2-\frac{1}{\phi\eta}\frac{\partial}{\partial\eta}\left(\eta\frac{\partial U}{\partial\eta}\right), \qquad (5)$$

$$\frac{1}{4}U\frac{\partial\theta}{\partial\xi}=\frac{1}{\eta}\frac{\partial}{\partial\eta}\left[\eta(\lambda_e+\lambda_d)\frac{\partial\theta}{\partial\eta}\right]+\frac{1}{4}E_k^*E_u^*Re^2U(1-B\theta). \qquad (6)$$

The equation of continuity may be represented in the integral form

$$2\int_0^1\eta U\,\mathrm{d}\eta=1. \qquad (7)$$

The relevant boundary conditions are

$$\eta=0:\ \partial U/\partial\eta=\partial\theta/\partial\eta=0,\ \eta=1:\ U=0,\quad \partial\theta/\partial\eta=0 \quad \text{for the adiabatic wall or} \quad \theta=1 \quad \text{for the isothermal wall.} \qquad (8)$$

3. Numerical methods

To determine the velocity and temperature profiles, the dimensionless momentum and energy equations were integrated numerically using a finite difference scheme. Variable grids in the axial direction were adopted to produce accurate velocity and temperature results. The grid size in the axial direction was very fine near the inlet and coarser downstream, but fine enough to yield accurate results. The dimensionless radius of the bed was divided into 300 equally spaced increments for the adiabatic boundary condition. However, for isothermal wall, the radius of the bed was divided into 331 unequally spaced increments and dimensionless mesh size $\Delta\eta$ was increased gradually from the wall toward the core region according to a geometric series defined by $\Delta\eta = \Delta\eta_0 a^{n-1}$ with $\Delta\eta_0 = 5\times 10^{-5}$ and $a = 1.018$. The diffusion terms were discretized using central difference formulas while the convection term in the energy equation was discretized by a backward difference scheme.

Since the Forchheimer term in the momentum equation is nonlinear, initial values of the velocity field were guessed at all the radial grid points for linearization and then this term was represented as the product of the unknown velocity and the guessed velocity. Finite difference equations derived from the momentum equation together with Eq. (7) constitute a set of simultaneous linear equations and were solved by Gaussian elimination to yield the velocity field for one iteration. The above process was repeated until convergence was achieved within a prescribed error. With the results of velocity in hand, the discretized form of the energy equation was solved to yield the temperature field.

The mixing-cup temperature was evaluated by

$$T_m = 2\pi \int_0^{r_0} T(r)u(r)r\,dr/\pi r_0^2 u_m, \tag{9}$$

and may be rewritten in the dimensionless form

$$\theta_m = 2\int_0^1 U\theta\eta\,d\eta. \tag{10}$$

For the adiabatic boundary condition, we can obtain the following analytical result as

$$\theta_m(\xi) = [1-\exp(-E_k^* E_u^* Re^2 B\xi)]/B + \exp[-E_k^* E_u^* Re^2 B\xi]. \tag{11}$$

Surprisingly, this expression indicates that the thermal conductivity of the solid does not affect the mixing-cup temperature at all. The local mean Nusselt number in the packed bed was defined by

$$Nu_m(\xi) = \ln[(1-\theta_0)/(1-\theta_m(\xi))]/4\xi. \tag{12}$$

4. Physical properties

(a) *Stagnant effective thermal conductivity*: It was evaluated utilizing Bruggeman's theory. According to this theory, λ_e may be given by

$$\lambda_e = \frac{k_e}{k_f} = (\kappa-1)\kappa^{1/3}\phi\left[\sqrt[3]{\frac{-1+\sqrt{A}}{2}} - \sqrt[3]{\frac{1+\sqrt{A}}{2}}\right] + \kappa, \tag{13}$$

where

$$A = 1 + \left(\frac{4}{27}\right)\phi^3\frac{(\kappa-1)^3}{\kappa^2}.$$

(b) *Lateral thermal dispersion conductivity*: K_d represents a degree of thermal transport due to the lateral mixing of local fluid streams within a packed bed and was given by the following expression [6]:

$$\lambda_d = k_d/k_f = 0.3519(1-\phi)^{2.3819} Pr\,Re\,U(\eta)/2\Gamma. \tag{14}$$

(c) *Porosity distribution function*: To account for the porosity variations near the wall, the following expressions [6] were utilized throughout the present study:

$$\begin{aligned}&\text{For } 0 \leqslant \zeta \leqslant 0.6,\\ &\quad \phi(\zeta) = 1 - 3.10036\zeta + 3.70243\zeta^2 - 1.24612\zeta^3.\\ &\text{For } 0.6 < \zeta \leqslant \Gamma,\\ &\quad \phi(\zeta) = -0.1865\exp(-0.22\zeta_1^{1.5})\cos(7.66\zeta_1) + 0.39,\end{aligned} \tag{15}$$

where $\zeta = (r_0 - r)/d_p$ and $\zeta_1 = \zeta - 0.6$.

5. Results and discussion

In order to examine the effect of viscous dissipation, calculations were performed for both adiabatic and isothermal wall conditions. Fig. 1 illustrates the variations of the mixing-cup temperature along the flow direction for the adiabatic boundary condition. The dimensionless bulk mean temperature increases monotonically with ξ. From this figure, it is clear that heat is locally generated due to viscous dissipation and the amount of generated heat increases with Γ and the Reynolds number. The agreement between the numerical results and the analytical predictions from Eq. (11) is so excellent that the difference between them is indiscernible.

Fig. 2 illustrates the comparison of Nu_m calculated for isothermal wall with the experimental data of Chennakesavan and Kuzuoka reported in [9]. The basic data necessary for the theoretical calculations such as Pr, κ, Γ and Re were taken from the same literature [9]. The solid lines represent Nu_m with viscous dissipation,

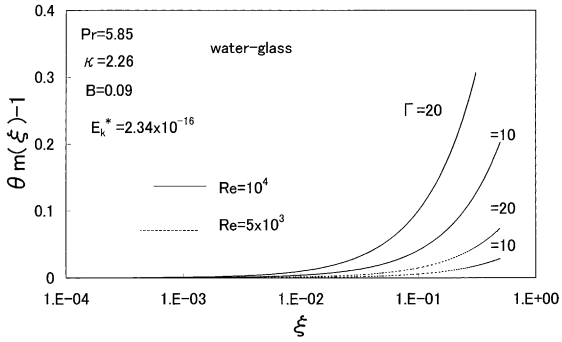

Fig. 1. Axial variations in the dimensionless mean temperature rise for the adiabatic wall.

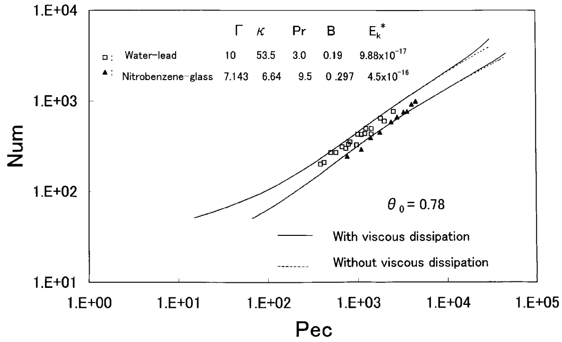

Fig. 2. Calculated and experimental mean Nusselt numbers for the isothermal wall.

while the broken lines represent those for without viscous dissipation. The experimental data are denoted by the symbols. It is seen that numerical results agree satisfactorily with the experimental ones, and this confirms the validity of the present model. It is also found that at Peclet numbers greater than about 10^4, the effect of viscous dissipation on mean Nusselt number is significant. This is caused by the fact that, as Peclet number is increased, local mean temperature is increased due to internal fluid friction within a packed bed and thus the value of local mean Nusselt number is raised. This tendency agrees with the case of heat transfer in empty tubes. A detailed parametric study of packed-bed heat transfer in the presence of viscous dissipation has been made in [10].

6. Conclusion

Thermally developing forced-convection heat transfer in cylindrical packed-bed, under the conditions of the adiabatic and isothermal boundaries were examined theoretically based on a two-dimensional, distributed parameter model taking account of the effects of non-Darcy, viscous dissipation, variable porosity and radial thermal dispersion. Special emphasis was given to the heat generation due to viscous dissipation in forced convection liquid flow. It was found that, in the case of the adiabatic boundary, the effect of internal heat generation associated with viscous dissipation increases with Γ and Reynolds number. For the isothermal boundary, the effect of viscous dissipation on heat transfer characteristics is negligible in the range of Peclet number less than about 10^4.

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